Thus, increasing $(r_2 - r_1)/\tau$ implies increasing $(r_2 - r_1)/d_v$. The graphs in Figure 1 show that f_v is increasing with increasing $(r_2 - r_1)/\tau$ and, hence, increasing $(r_2 - r_1)/d_v$.

In Figure 2, however, $\Delta r^* = (r_2 - r_1)/\tau$ is kept constant and $d_v^* = d_v/\tau$ is varied, which amounts to varying d_v while keeping $r_2 - r_1$, and τ constant. This, of course, is the same thing as varying both $r_2 - r_1$, and τ in the same proportion while keeping d_v constant. The graphs in Figure 2 indicate an increase of f_v with decreasing d_v/τ, that is, with increasing τ and $r_2 - r_1$, with d_v kept constant. (Note that τ and $r_2 - r_1$ are always increased in the same proportion so as to keep the ratio $(r_2 - r_1)/\tau$ constant.) It is evident from the diagram that under these rather special conditions the effect of $r_2 - r_1$ (causing f_v to increase) outweighed that of τ (causing f_v to decrease). This behavior might appear to be contradicting the experimental results of Batra. These results, however, cannot be considered general for several obvious reasons. Because of the special way in which the effects of the two independent variables $r_2 - r_1$ and τ are combined, one cannot even conclude that the absolute value of $\partial f_v/\partial[(r_2 - r_1)/d_v]$ is greater than that of $\partial f_v/\partial(\tau/d_v)$, as the simultaneous changes in $(r_2 - r_1)/d_v$ and in τ/d_v are not in general of the same magnitude. The results would be much easier to interpret if in Figure 2 $(r_2 - r_1)/d_v$ had been kept constant and τ/d_v had been varied.

Payatakes et al., however, appear to have also performed calculations using some values of the parameters in Batra's experiments, the results of which were plotted in their Figure 10. It is evident from this diagram that their calculations have predicted decreasing f_v with increasing τ/d_v and $(r_2 - r_1)/d_v$ also for the specific conditions of some of Batra's experiments. This, of course, amounts to a direct conflict between the experimental measurements of Batra and the calculated results of Payatakes et al. Whether this basic disagreement is due to the different geometries of the flex-tubes and the model, or is caused by some flaw in the model or in the experimental work, cannot be decided at this time. It is a fact, however, that the great mass of data gathered by Batra on flextubes consistently gave the same dependence of f_v on τ and $r_2 - r_1$, for constant d_v.

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The authors are grateful to H. Ertl for helpful discussions.

NOTATION

- d_v = volumetric diameter of a uniform periodically constricted tube
- d_v^* = dimensionless volumetric diameter
- f_v = friction factor
- r_1 = radius of constriction
- r_2 = maximum radius
- r_1^* = dimensionless radius of constriction (r_1/τ)
- r_2^* = dimensionless maximum radius (r_2/τ)

Greek Letters

- Δr = $r_2 - r_1$, amplitude
- Δr^* = $r_2^* - r_1^*$, dimensionless amplitude
- τ = length of segment of a periodically constricted tube

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Further Work on the Flow Through Periodically Constricted Tubes—A Reply

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In the preceding note (1973), Dullien and Azzam raised certain questions concerning the earlier work of Payatakes et al. (1973). In general, their major criticisms center around two issues: (1) the interpretation of the comparison between the experimental results of Batra (1969) and the numerical solution obtained by Payatakes et al. (1973), and (2) the correct assessment of the various geometry factors affecting the pressure drop (friction factor) in periodically constricted tubes. A brief discussion of these two issues is given here. In addition, questions concerning the adequacy and applicability of the data obtained with the use of extensible flex-tubes in the study of flow through periodically constricted tubes in general are presented. The question of the adequacy and applicability of the data, in addition to being germane to the present discussion, is of some fundamental importance to the flow problem at hand.

Dullien and Azzam took exception to the criticism of Payatakes et al. (1973) concerning the validity of the statement of Batra et al. (1970) and Dullien and Batra (1970), namely, "The value of friction factor increases with decreasing wavelength-to-diameter ratios (<0.5) by as much as 120% of the uniform tube value (wave amplitude-to-diameter ratios were in the range 0.13 to 0.25)." By its omission to specify the mode of variation of $(r_2 - r_1)/d_v$, the above statement suggests that within the given range of amplitude-to-diameter ratio, f_v depends mainly on the value of τ/d_v (or $d_v^* = d_v/\tau$). This statement is found to be at variance with the numerical solution as shown in Table 1. In Table 1, for a few cases chosen randomly, values of $f_v(N_{Re})_v$ for fixed d_v^* (or τ/d_v) but different values of $(r_2 - r_1)/d_v$ are given as calculated by the numerical method developed in Payatakes et al. (1973). As can be seen, the effect of $(r_2 - r_1)/d_v$ is comparable to that of d_v^*. Furthermore, as shown in Tables 2 and 3, $f_v(N_{Re})_v$ can be either an increasing, or decreasing function of d_v^* when d_v^* and $\Delta r/d_v$ are selected from the range of Batra's experiments, but independently of each other. It is clear that whatever validity the statement of Batra et al. may hold for the experimental results obtained with the use of extensible flex-tubes, it certainly cannot be applied to periodically constricted tubes in general.

TABLE 1. EFFECT* OF $\Delta r/d_v$ ON $f_v(N_{Re})_v$

d_v^*	Δr^*	$\Delta r/d_v$	$2r_1^* = 2r_3^*$	$2r_2^*$	N	$f_v(N_{Re})_v$
2.0	0.500	0.25	1.46941	2.46941	9	37.76
2.0	0.260	0.13	1.73177	2.25177	9	22.25
2.2	0.286	0.13	1.90495	2.47695	9	22.86
2.2	0.550	0.25	1.61635	2.71635	9	39.17

* Wall geometry described by Equation (29) in Payatakes et al. (1973).

TABLE 2. EXAMPLE* OF $f_v(N_{Re})_v$ DECREASING WITH INCREASING d_v^* FOR LOW REYNOLDS NUMBERS

d_v^*	Δr^*	$\Delta r/d_v$	$2r_1^* = 2r_3^*$	$2r_2^*$	N	$f_v(N_{Re})_v$
2.0	0.500	0.25	1.46941	2.46941	9	37.76
2.1	0.399	0.19	1.68251	2.48051	9	28.71
2.2	0.286	0.13	1.90495	2.47695	9	22.86

* Wall described by Equation (29) in Papatakes et al. (1973).

TABLE 3. EXAMPLE* OF $f_v(N_{Re})_v$ INCREASING WITH INCREASING d_v^* FOR LOW REYNOLDS NUMBERS

d_v^*	Δr^*	$\Delta r/d_v$	$2r_1^* = 2r_3^*$	$2r_2^*$	N	$f_v(N_{Re})_v$
2.0	0.260	0.13	1.73177	2.25177	9	22.25
2.1	0.399	0.19	1.68251	2.48051	9	28.71
2.2	0.550	0.25	1.61635	2.71635	9	39.17

* Wall described by Equation (29) in Papatakes et al. (1973).

It may also be pertinent to note that the statement in the note by Dullien and Azzam, namely, that Batra et al. (1970) showed that "the effect of τ/d_v on f_v far outweighed the effect of $(r_2 - r_1)/d_v$" is not justified since it implies that in the range of experimentation the effect of $(r_2 - r_1)/d_v$ on f_v is negligible compared to the effect of τ/d_v. This conclusion cannot be drawn from the actual statement of Batra et al. "the effect of τ/d_v on f_v outweighs the effect of $(r_2 - r_1)/d_v$" which simply indicates that in the case of their experimental results the effect of τ/d_v on f_v is more pronounced than that of $(r_2 - r_1)/d_v$.

From the numerical analysis it is obvious that the relevant geometry parameters are $d_v^* = d_v/\tau$, $\Delta r^* = (r_2 - r_1)/\tau$ and the geometry of the constriction. The constriction geometry can be specified by defining the form of the equation which gives the radial distance of the tube wall from the tube axis as a function of axial position, as, for example, Equation (29) in Payatakes et al. (1973). Payatakes et al. give examples of numerical calculations, for a given constriction geometry, which indicates that for constant d_v^* the product $f_v(N_{Re})_v$ is an increasing function of Δr^* and also that for constant Δr^* the product $f_v(N_{Re})_v$ is a decreasing function of d_v^*. Batra et al. (1970) used as independent dimensionless groups τ/d_v and $(r_2 - r_1)/d_v$, which are, of course, equivalent, being equal to $1/d_v^*$ and $\Delta r^*/d_v^*$, respectively. More recently, numerical values of $f_v(N_{Re})_v$ for constant $(r_2 - r_1)/d_v$ and varying d_v^* were also obtained using the method developed by Payatakes et al. and are presented in Table 4. As can be seen, for $(r_2 - r_1)/d_v =$ constant, the product $f_v(N_{Re})_v$ is an increasing function of d_v^*.

In the preceding note, Dullien and Azzam raised questions concerning the generality of the dependence of f_v on d_v^* for constant Δr^*, as presented in Figure 9 of Payatakes et al. (1973) (Figure 2 of preceding note). Their argument is misleading. Dullien and Azzam contend that the behavior of f_v due to the variation of d_v^* for constant Δr^* as shown in this figure cannot be considered general for several obvious reasons (unspecified). Their objection apparently rests upon the argument that the results as shown in their Figure 2, when interpreted in a particular manner, indicate the effect of $(r_2 - r_1)$ outweighs that of τ, which is contrary to their experimental data. However, they have overlooked a more straightforward interpretation, which shows that the behavior of f_v displayed in Figure 2 is indeed expected in general. The periodically constricted tube can be considered as a geometric entity obtained through the revolution of a periodic curve around a parallel axis placed at a certain distance away from the curve. If one considers two cases which correspond to the same curve but placed at two different distances away from the same axis, the two resulting periodically constricted tubes would have the same values of τ and $(r_2 - r_1)$ but different values of d_v. Under such conditions, one could expect that the friction factor f_v of the tube with larger d_v should be less than that of the tube with smaller d_v; that is, $f_v(N_{Re})_v$ decreases with increasing d_v, which under the condition of constant τ and $(r_2 - r_1)$, becomes identical to that shown in Figure 2.

The numerical solution clearly suggests that for a periodically constricted tube with specific description of the shape of the constriction, the product $f_v(N_{Re})_v$ is a function of two geometry parameters: d_v/τ and $(r_2 - r_1)/\tau$ or their equivalents [for example, $(r_2 - r_1)/d_v$ and τ/d_v, or r_1/τ and r_2/τ]. Under certain restricted conditions, one

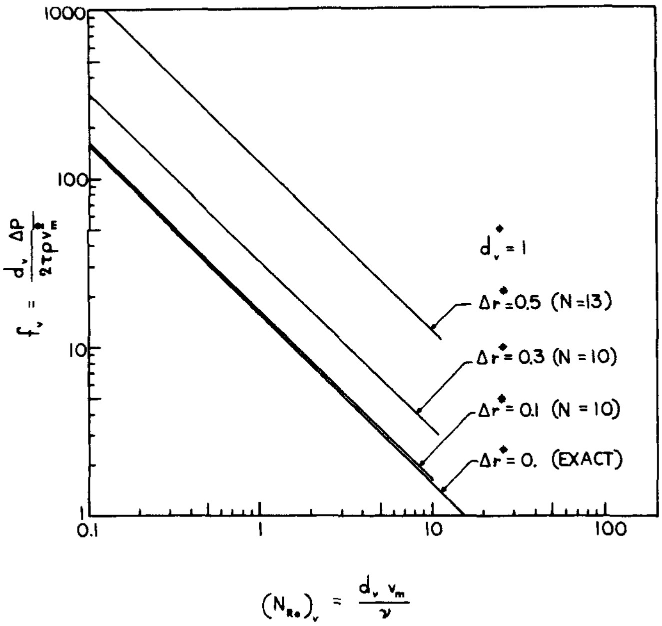

Fig. 1. Effect of the variation of the amplitude on the dimensionless pressure drop for constant volumetric diameter (after Payatakes et al., 1973).

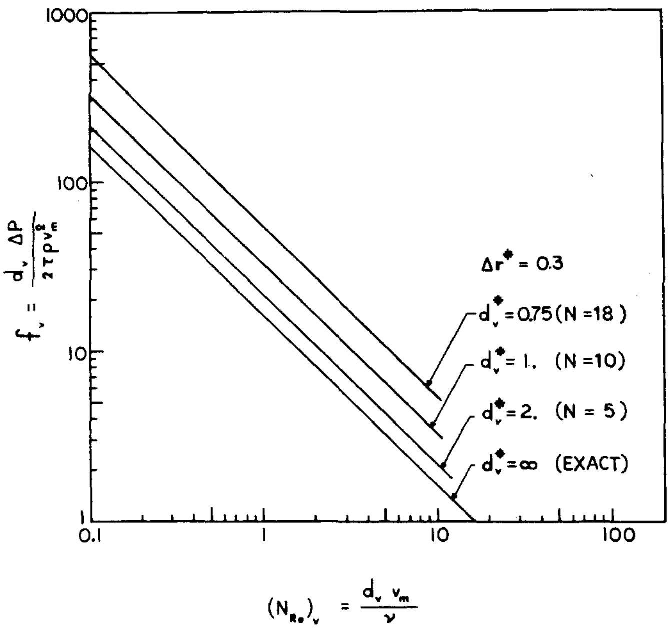

Fig. 2. Effect of the variation of the volumetric diameter on the friction factor for constant amplitude (after Payatakes et al., 1973).

may speak of the relative importance of one dimensionless parameter over the other if the exact solution or extensive experimental data are available. However, it would be confusing and indeed improper to think in terms of the relative significance of the three individual physical variables because they cannot be considered as independent variables. In other words, one cannot make a meaningful assessment of the relative importance of $(r_2 - r_1)$ and τ on f_v from experimental observations unless the experiments are conducted under the condition of constant d_v for all cases.

To compare Batra's data with our numerical solution, we assumed that the correlation obtained by Batra et al. (1970) was intended to be a generalized correlation for tubes with periodic constrictions. The use of extensible flex-tubes introduced additional constraints, but the fact that these constraints were not incorporated by Batra et al. (1970) into their final correlation presumably implies that they were considered relatively unimportant. The comparison of our numerical solution with the data by

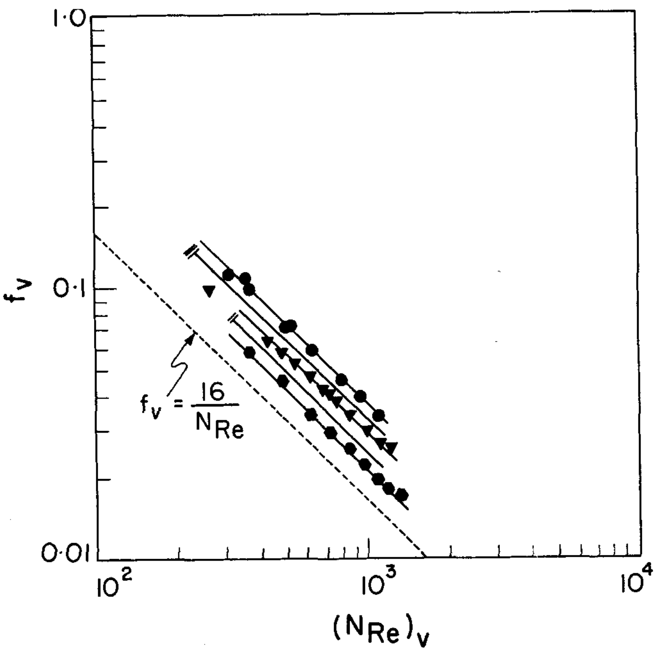

Fig. 3. Friction factor as a function of Reynolds number for ¼-in wavy tube with wavelength-to-diameter ratio as parameter (Batra, 1969). Flowing Fluid—Water

Table	τ/d_v	$(r_2 - r_1)/d_v$
● 13, 14	0.4495	0.171
||| 11, 12	0.4528	0.180
▲ 10	0.4566	0.186
|| 6, 7, 8, 9	0.4602, 0.4610	0.195
⬡ 5	0.4654	0.210

Table 4. Examples* of Dependence of $f_v(N_{Re})_v$ or d_v^* with $(r_2 - r_1)/d_v$ Constant for Low Reynolds Numbers

d_v^*	Δr^*	$\Delta r/d_v$	$2r_1^* = 2r_3^*$	$2r_2^*$	N	$f_v(N_{Re})_v$
1.50	0.150	0.1	1.34635	1.64635	12	19.10
1.00	0.100	0.1	0.89757	1.09757	15	18.03
0.75	0.075	0.1	0.67318	0.82318	20	17.66
0.50	0.050	0.1	0.44878	0.54878	25	16.96
1.50	0.450	0.3	1.01685	1.91685	14	44.63
1.00	0.300	0.3	0.67790	1.27790	20	38.09
0.75	0.225	0.3	0.50842	0.95842	25	34.22
0.50	0.150	0.3	0.33895	0.63895	25	29.55

* Wall described by Equation (29) in Papatakes et al. (1973).

Table 5. Numerical Results* Corresponding to Tables 5, 10, 13, and 14 in Batra et al. (1970)

d_v^*	Δr^*	$\Delta r/d_v$	$2r_1^* = 2r_3^*$	$2r_2^*$	N	$f_v(N_{Re})_v$
2.14869	0.45122	0.210	1.67433	2.57677	9	31.73
2.19010	0.40736	0.186	1.76426	2.57898	9	28.64
2.22469	0.38042	0.171	1.82842	2.58926	9	27.25

* Wall described by Equation (29) in Papatakes et al. (1973).

TABLE 6. EFFECT* OF CONSTRICTION GEOMETRY ON $f_v(N_{Re})_v$ OF PERIODICALLY CONSTRUCTED TUBES AT LOW $(N_{Re})_v$

d_v^*	Δr^*	$\Delta r/d_v$	$2r_1^* = 2r_3^*$ (1)	$2r_1^* = 2r_3^*$ (2)	$2r_2^*$ (1)	$2r_2^*$ (2)	$f_v(N_{Re})_v$ (1)	$f_v(N_{Re})_v$ (2)
2.14869	0.45122	0.210	1.68162	1.67433	2.58406	2.57677	29.82	31.73
2.19010	0.40736	0.186	1.77008	1.76426	2.58480	2.57898	27.23	28.64
2.22469	0.38042	0.171	1.83340	1.82842	2.59424	2.58926	25.81	27.25

(1) 1st degree of polynomial type of constriction (saw-tooth geometry).
(2) 3rd degree polynomial type of constriction [Equation (29) in Payatakes et al. (1973)].
* $N = 9$.

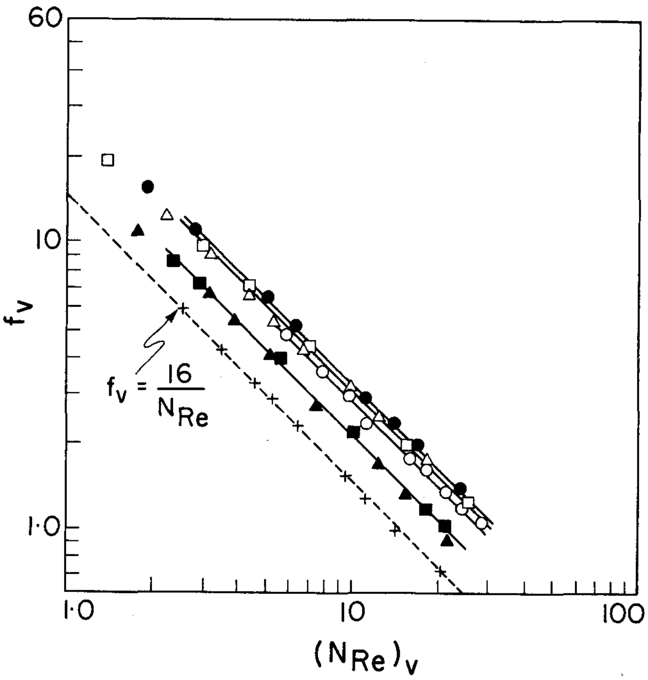

Fig. 4. Friction factor as a function of Reynolds number for ¼-in wavy tube with wavelength-to-diameter ratio as parameter (Batra, 1969).

Flowing Fluid—Corn Syrup Solution

Table	λ/d_v	$(r_2 - r_1)/d_v$
+ 4	straight tube	
▲ 15	0.4639	0.233
■ 16	0.4672	0.213
○ 17	0.4505	0.173
△ 18	0.4489	0.171
□ 19	0.4456	0.163
● 20	0.4440	0.160

Batra et al. raised questions about the generality of their correlation and this was stated in our work. As shown above, the arguments set forth in the note by Dullien and Azzam, in our opinion, do not refute this conclusion.

To further illustrate this point, values of $f_v(N_{Re})_v$ for low $(N_{Re})_v$ corresponding to the various geometry parameters listed in Tables 5, 10, 13, and 14 of Batra et al. (1970) were obtained, using the numerical algorithm developed by Payatakes et al. (1973), and are summarized in Table 5. It is interesting to note that $f_v(N_{Re})_v$ decreases with increasing d_v^*, which is contrary to the experimental observation of Batra et al. A tentative explanation for this discrepancy is the following:

A factor which has not been carefully considered in the study of flow through periodically constricted tubes, but may have significant effect, is the geometry of the constriction. Our earlier calculations were based on the assumption that the constriction can be described by a third degree polynomial [Equation (29) in Payatakes et al. (1973)]. As it was also pointed out in Payatakes et al. (1973), comparison of calculated values of f_v [using Equation (29) in Payatakes et al. to describe the wall] and Batra's (1969) experimental values are not entirely rigorous since the walls of the tubes used by Batra in his experiments were neither completely periodic, nor exactly defined, and had substantial irregularities. More recently we have made a few calculations assuming that the constrictions are sawtooth-like. A comparison of these two cases is given in Table 6 which shows the significant difference in the values of $f_v(N_{Re})_v$ for the two types of constriction while all other parameters (d_v^*, Δr^*, and therefore $\Delta r/d_v$) are identical. Furthermore, this constriction geometry effect appears to be as important as those of d_v^* and Δr^*.

From the foregoing discussion, it appears that the important parameters in the flow through periodically constricted tubes are d_v^* and Δr^* (or $\Delta r/d_v$) and the constriction geometry. Any generalized correlation to be developed in the future must have all these factors taken into account. The use of extensible flex-tubes in conducting experiments, while convenient in certain aspects, has some serious limitations since its use does not allow independent variations of the geometry parameters. In addition, there is, in all probability, a gradual change of the constriction geometry as the tubes are stretched, and this change may not be easily assessed quantitatively. From these considerations, it follows that it would be rather difficult to establish a truly generalized correlation for flow through periodically constricted tubes based on experimental results obtained using extensible flex-tubes.

ACKNOWLEDGMENT

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NOTATION

Symbols are identical to those in Payatakes et al. (1973).

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